ON INVARIANT IDEALS OF REPRESENTATION RINGS OF SEMISIMPLE GROUPS

To any semisimple group, one can associate its weight lattice $\tilde{\Lambda}$, the set of simple weights $\varpi_1, \ldots, \varpi_n$, and the Weyl group W acting on $\tilde{\Lambda}$. One can consider the Laurent polynomial rings $\mathbb{Q}[\tilde{\Lambda}]$ and $\mathbb{Z}[\tilde{\Lambda}]$ (the monomial corresponding to $\lambda \in \tilde{\Lambda}$) will be denoted by e^λ and the *augmented orbit polynomials* $p_i = -|W\varpi_i| + \sum_{\lambda \in W\varpi_i} e^\lambda$. These polynomials generate ideals $\tilde{I} \subset \mathbb{Z}[\tilde{\Lambda}]$ and $\tilde{I_{\mathbb{Q}}} \subset \mathbb{Q}[\tilde{\Lambda}]$.

One can also consider the root lattice $\Lambda \subseteq \tilde{\Lambda}$ and the corresponding Laurent polynomial subrings $\mathbb{Z}[\Lambda] \subseteq \mathbb{Z}[\tilde{\Lambda}]$ and $\mathbb{Q}[\Lambda] \subseteq \mathbb{Q}[\tilde{\Lambda}]$.

If certain (not very strong in the case of $\mathbb{Q}$, and very strong in the case of $\mathbb{Z}$) conditions on Λ and $\tilde{\Lambda}$ are satisfied, I will explain how to find the intersections $\tilde{I} \cap \mathbb{Z}[\Lambda]$ and $\tilde{I_{\mathbb{Q}}} \cap \mathbb{Q}[\Lambda]$.