Locally nilpotent derivations of rings with roots adjoined

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Abstract. Many current problems in affine algebraic geometry involve varieties whose coordinate rings take the form $B = R[z]$, where R is an affine ring and $z^n \in R$ for an integer $n \geq 2$. The most prominent examples are the Russell cubic threefold $X \subset \mathbb{A}^4$, defined by $x + x^2y + z^2 + t^3 = 0$, and the Pham-Brieskorn varieties $V \subset \mathbb{A}^n$, defined by $x_1^{e_1} + \cdots + x_n^{e_n} = 0$ for integers $e_i \geq 2$. This talk will discuss joint work with Lucy Moser-Jauslin initiated in the summer of 2009. By studying the locally nilpotent derivations (LNDs) of B relative to those of R, we reveal a surprisingly effective general approach to understanding large classes of these varieties. In particular, our work features: (1) criteria to determine that certain rings are rigid; (2) a concise new proof that the Russell cubic X is not isomorphic to affine space; (3) proofs for rigidity for a large family of Pham-Brieskorn threefolds and related threefolds, some of which use recent results of Daigle; and (4) a generalization of Kolhatkar's theorem regarding homogeneous LNDs of polynomial rings. A number of important open problems will also be discussed.